Section 9.2

The Laws of Exponents
If b and c are positive and x and y are any real numbers, then the following laws hold:

a) $b^x b^y = b^{x+y}$

b) $\frac{b^x}{b^y} = b^{x-y}$

c) $\frac{1}{b^x} = b^{-x}$

d) $b^0 = 1$

e) $(b^x)^y = b^{xy}$

f) $(bc)^x = b^x c^x$

g) $\left(\frac{b}{c}\right)^x = \frac{b^x}{c^x}$

Exponential Function
An exponential function has the form

$$f(x) = Ab^x \qquad \text{or} \qquad y = Ab^x$$

where A and b and constants with $A \neq 0$ and b positive. We call b the base of the exponential function. A quantity experiences exponential growth if $y = Ab^t$ with $b > 1$. It experiences exponential decay if $y = Ab^t$ with $0 < b < 1$.

Compound Interest
If an amount (present value) P is invested for t years at an annual rate of r, and if the interest is compounded (reinvested) m times per year, then the future value A is

$$A = P\left(1 + \frac{r}{m}\right)^{mt}$$

A special case is interest compounded once a year: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

The Number e and Continuous Compounding
The number e is the limiting value of the quantities $\left(1 + \frac{1}{m}\right)^m$ as m gets larger and larger, and has the value 2.71828182845904523536 …
If $\$P$ is invested at an annual interest rate r compounded continuously, the accumulated amount after t years is

$$A(t) = Pe^{rt}$$

Exponential Functions: Alternative Form

We can write any exponential function in the following alternative form:

$$f(x) = Ae^{rx}$$

where A and r are constants. If r is positive, f models exponential growth; if r is negative, f models exponential decay.

Problem 1. The values of two functions, f and g, are given in a table. One, both, or neither of them may be exponential. Decide which, if any, are exponential, and give the exponential model for those that are.

x	-2	-1	0	1	2
$f(x)$	0.5	1.5	4.5	13.5	40.5
$g(x)$	8	4	2	1	$\frac{1}{2}$

Problem 2. Model the data using an exponential function $f(x) = Ab^x$.

x	0	1	2
$f(x)$	10	30	90

Problem 3. Find the equations for the exponential functions that pass through the pairs of points given in the following exercises. (Round coefficients to four decimal places when necessary.)

a) $(2, 45), (4, 405)$

b) $(2, 3),\ (6, 2)$

Problem 4. $f(t)$ is the value after t years of a \$5,000 investment earning 10% interested compounded continuously. Write the exponential function $f(t) = Ae^{rt}$ that models this situation.

Problem 5. A bacteria culture starts with 1,000 bacteria and doubles in size every 3 hours. Find an exponential model for the size of the culture as a function of time t in hours and use the model to predict how many bacteria there will be after 2 days.

Problem 6. In 2004, the Northern Rock Bank in the United Kingdom offered 4.76% interest on its online accounts, with interest reinvested annually. Find the associated exponential model for the value of a \$4000 deposit after t years. Assuming this rate of return continued for four years, how much would a deposit of \$4000, made at the beginning of 2004, be worth at the start of 2008?

Problem 7. After several drinks, a person has a blood alcohol level of 200 mg/dl (milligrams per deciliter). If the amount of alcohol in the blood decays exponentially, with one fourth being removed every hour, find the person's blood alcohol level after four hours.

Problem 8. World population was estimated at 2.56 billion people in 1950 and 6.4 billion people in 2004.

a) Use these data to give an exponential growth model showing the world population P as a function of t in years since 1950. Round coefficients to five decimal places.

b) Assuming the exponential growth model from part (a), estimate the world population in 1000 A.D.

Homework for this section: Read the section and watch the videos/tutorials. Then do these problems in preparation for the quiz: #9, 23, 33, 47, 62, 63, 67 (on Excel), 79, 84, 95